# Electrostatic effects and evaporation near apparent contact lines

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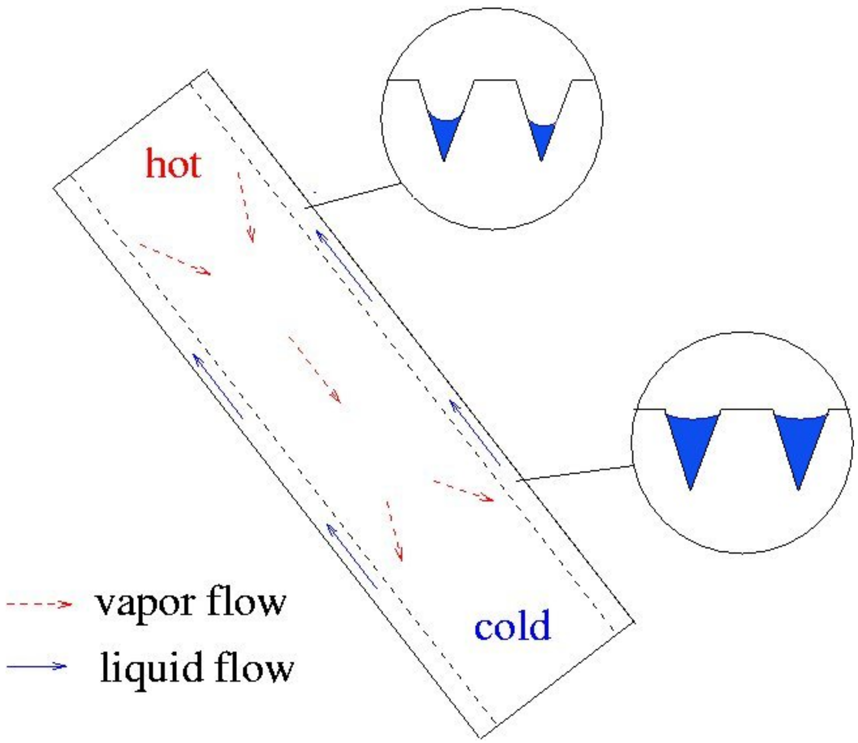
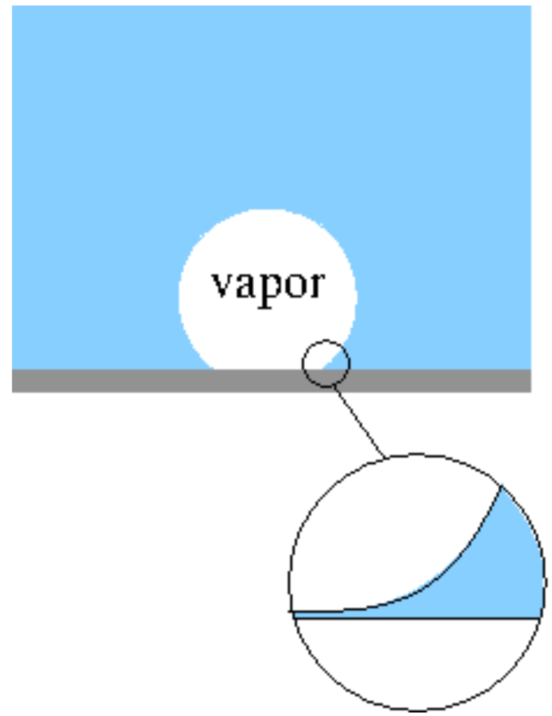
# Motivating applications

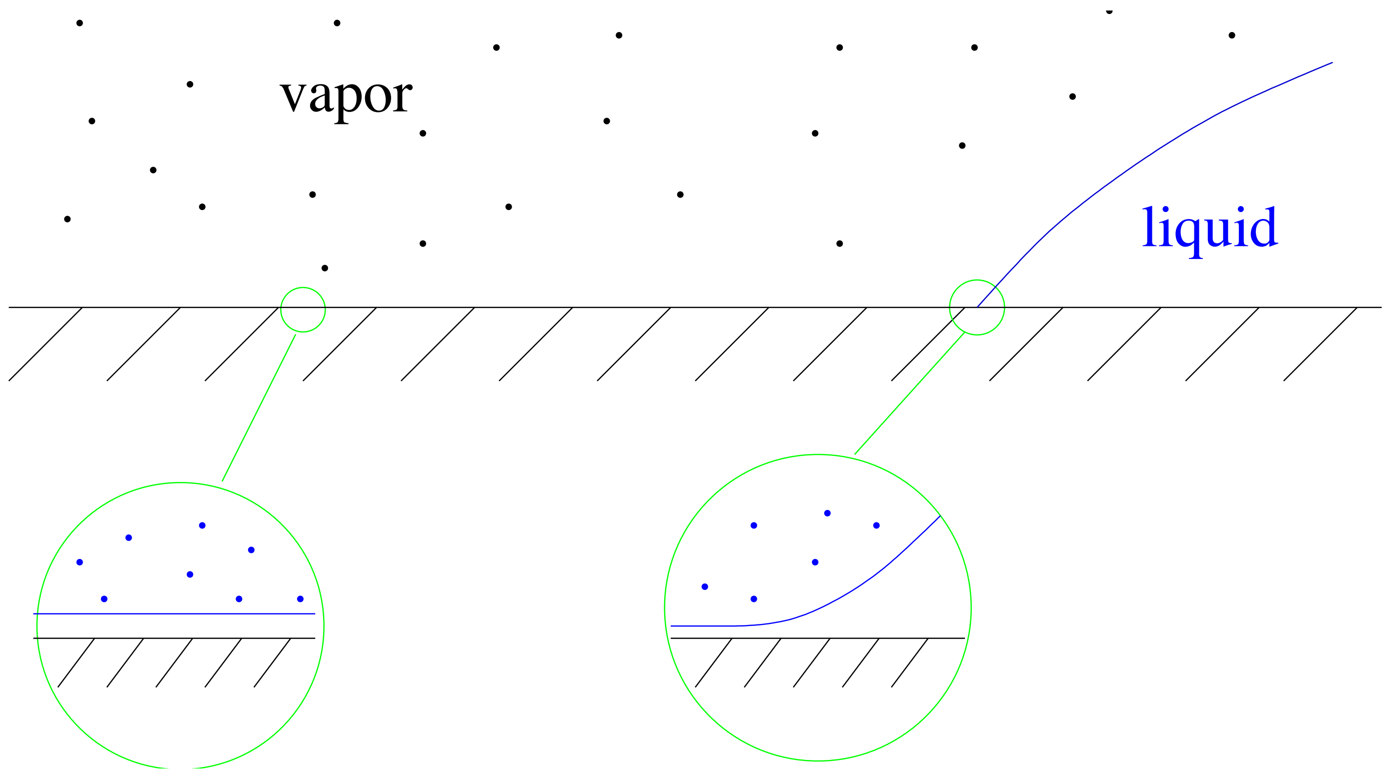
## Micro heat pipes




## Pool boiling



# Apparent contact lines

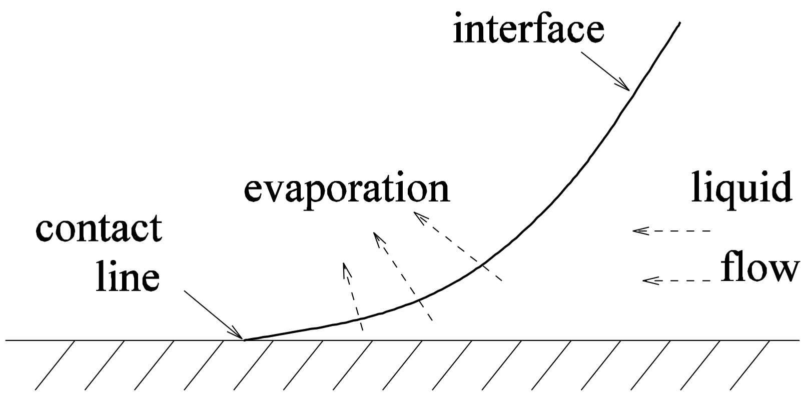



- “Dry” area is covered by ultra-thin film
- Apparent contact line is a transition zone

# Models of evaporation

Heat flux:

$$J^* = \frac{k\Delta T - \delta^*(\Delta P + \Pi)}{H}$$




Solutions with van der Waals disjoining pressure:

Potash & Wayner (1972), Moosman & Homsy (1980), Morris (2001), Ajaev et al. (2002), …

Electrostatic effects - ?

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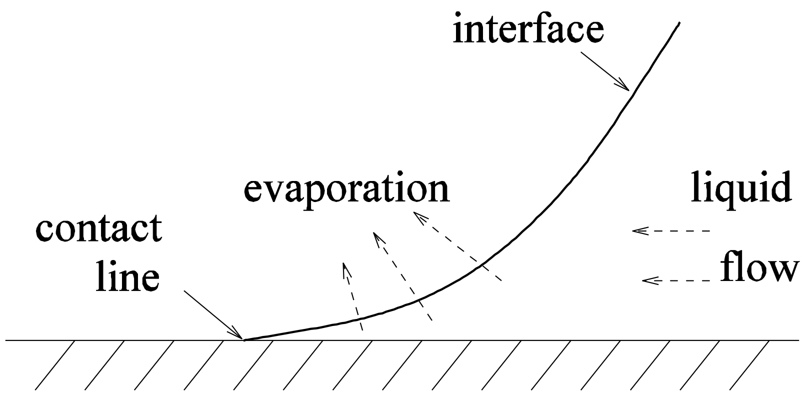


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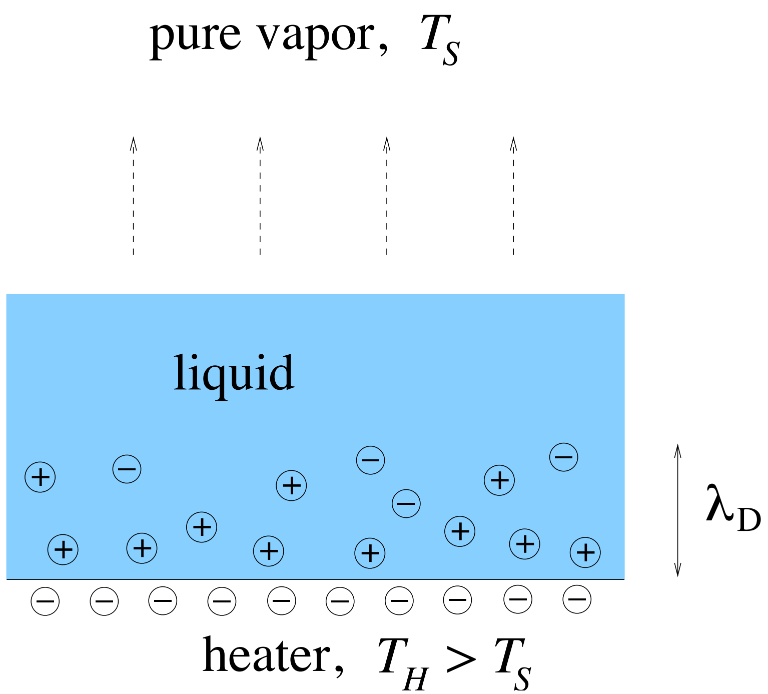
Electrostatic effects - ? Mazzoco & Wayner (1999)

# Electric field in the film




$$\psi_{yy} = \kappa^2 \sinh \psi$$

$$\psi(x,0,t) = \psi_0$$

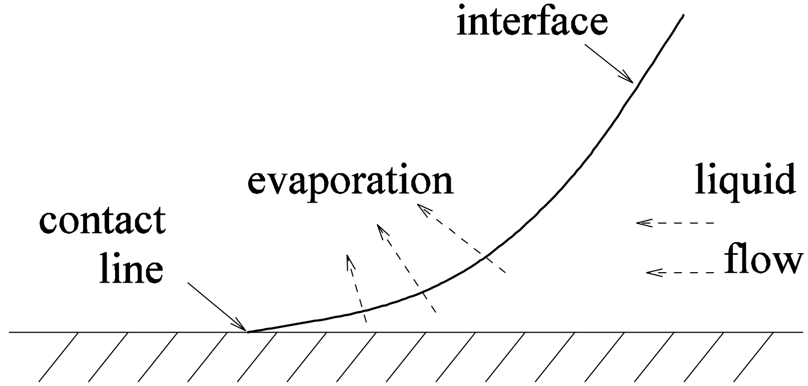
$$\psi_y(x,h,t) = \kappa q$$

$$\lambda_D = \left( \frac{2n_0 e^2}{\varepsilon k_B T} \right)^{-1/2}$$

Parameters:

$$\kappa = \frac{d}{\lambda_D}, \quad \psi_0, \quad q$$

# Evaporation and viscous flow




- Lubrication-type velocity profile $(d << L)$
- Uniformly heated substrate
- Small capillary number
- One-sided model of evaporation

# Non-dimensional model

Evaporative flux:

$$J = \frac{\delta\left(\frac{1}{2}\kappa^2 Q q^2 - h_{xx} - \kappa^2 Q \cosh\tilde{\psi} - \frac{\alpha}{h^3}\right) + T_0}{K + h}$$

Equation for thickness:

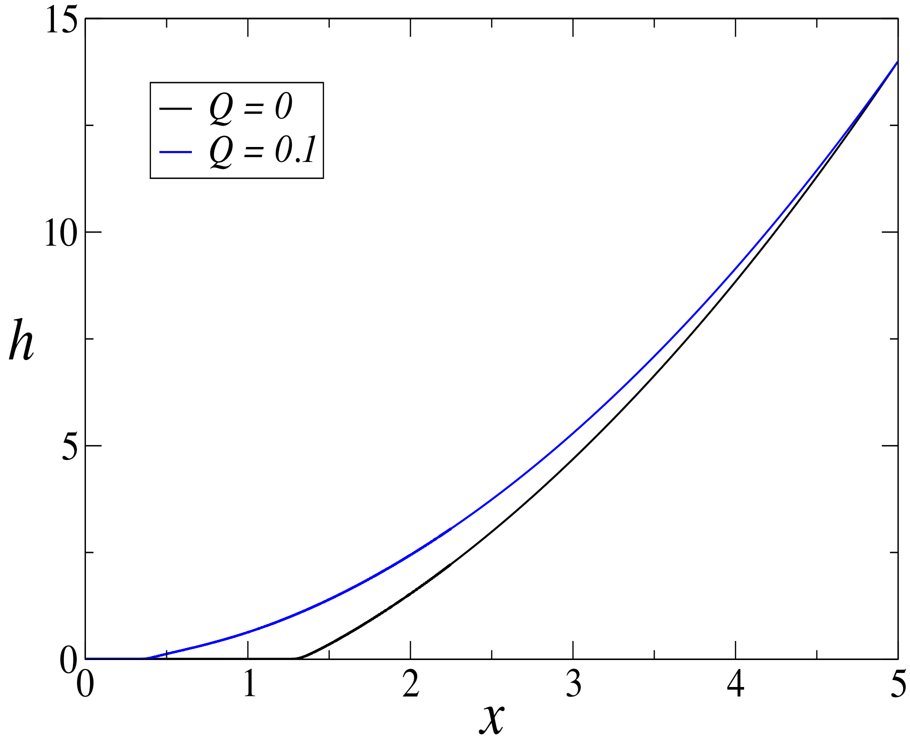
$$h_t + J + \frac{1}{3}\left[h^3\left(h_{xx} + \kappa^2 Q \cosh\tilde{\psi} + \frac{\alpha}{h^3}\right)_x\right]_x - \frac{1}{2}\kappa Q q\left[h^2 \tilde{\psi}_h h_x\right]_x = 0$$

Key parameters: $\kappa,\quad \psi_0,\quad q,\quad Q = \frac{\varepsilon}{\sigma R_0}\left(\frac{k_B T_S}{e}\right)^2,\quad T_0,\quad \alpha,\quad K,\quad \delta$

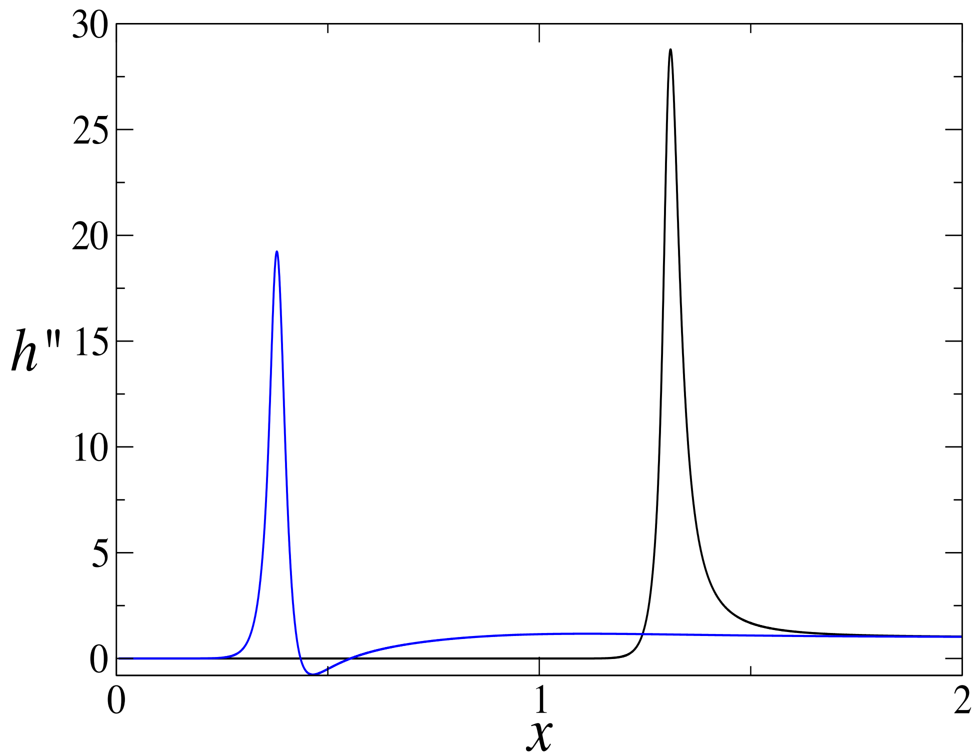
# Local liquid-vapor interface shapes

Consider steady-state solutions: $h_t = 0$

Film thickness




Curvature




$$q = 0.1, \quad \psi_0 = -5, \quad T_0 = 0.1, \quad q = -0.04, \quad \kappa = 3, \quad K = 0.1, \quad \delta = 10^{-4}, \quad \alpha = 10^{-3}$$

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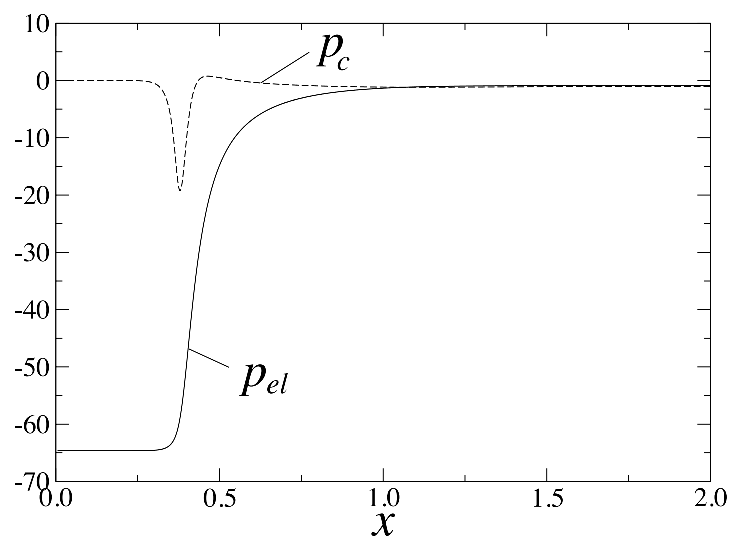

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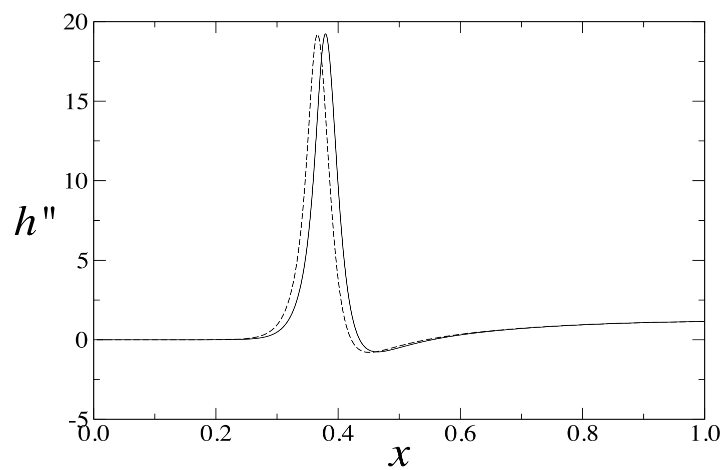
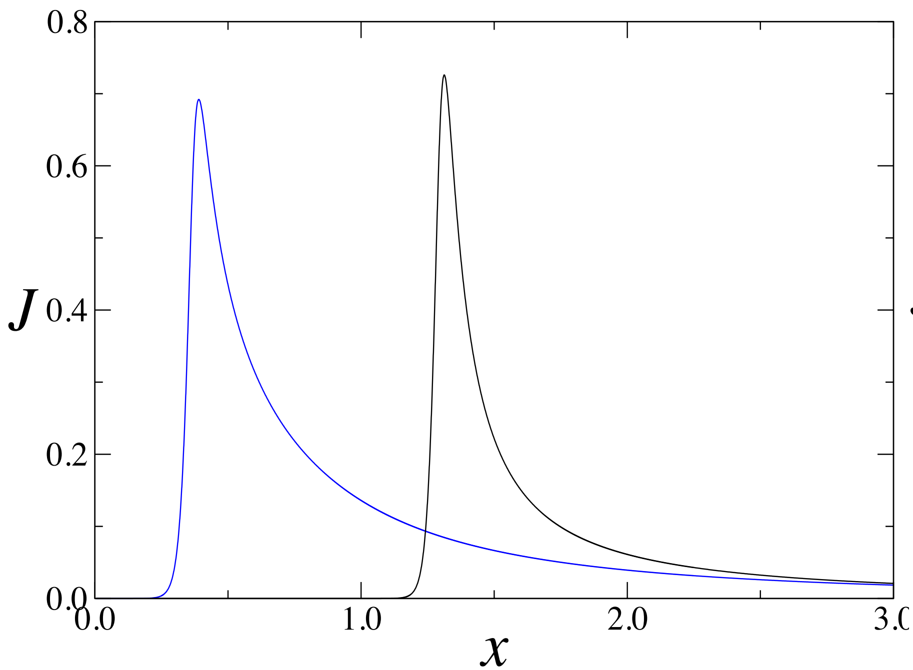
- Shear stress balance modification

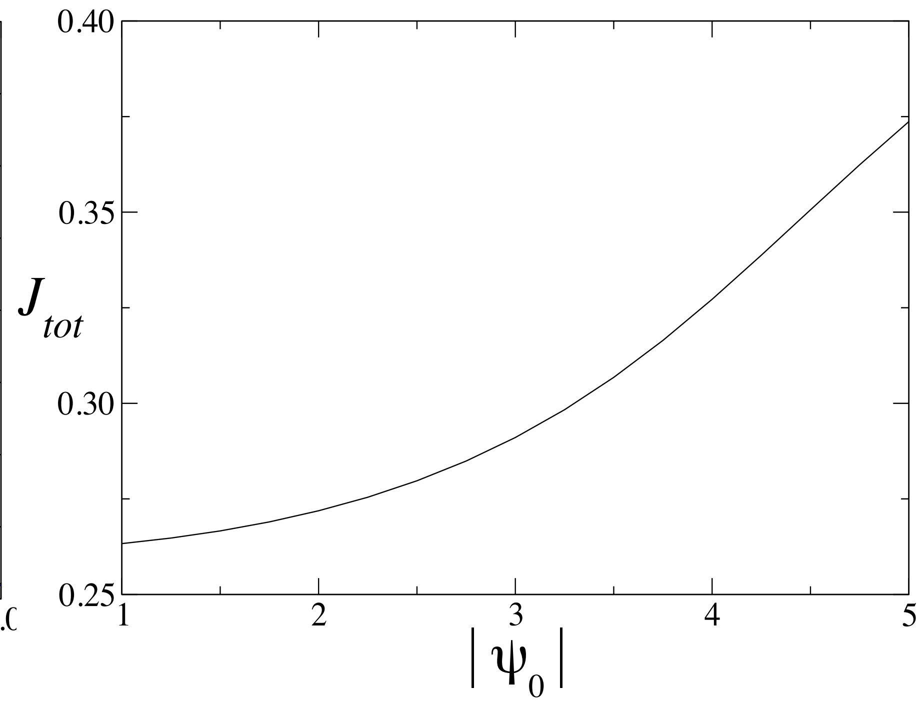


# Evaporation rates

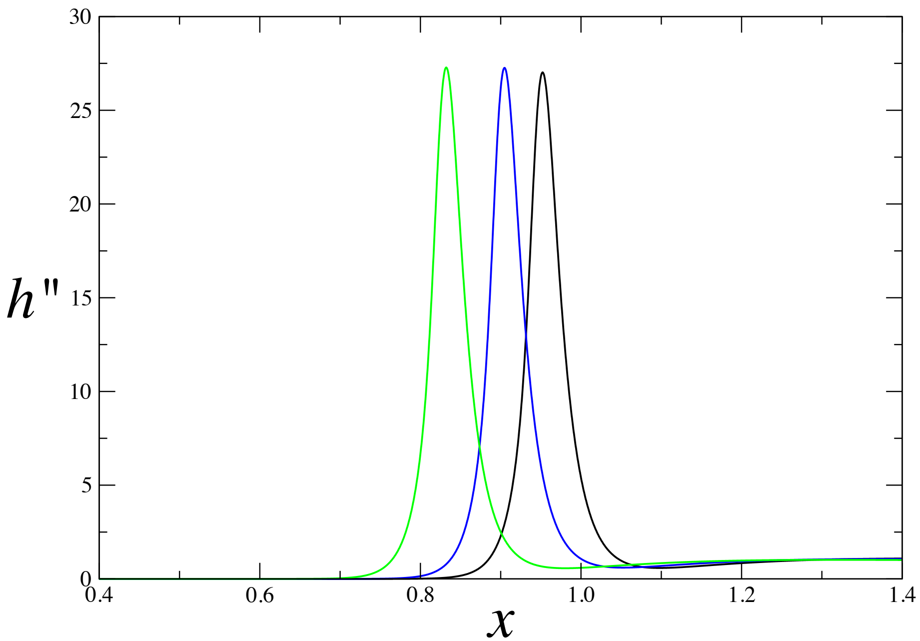
Local flux profile

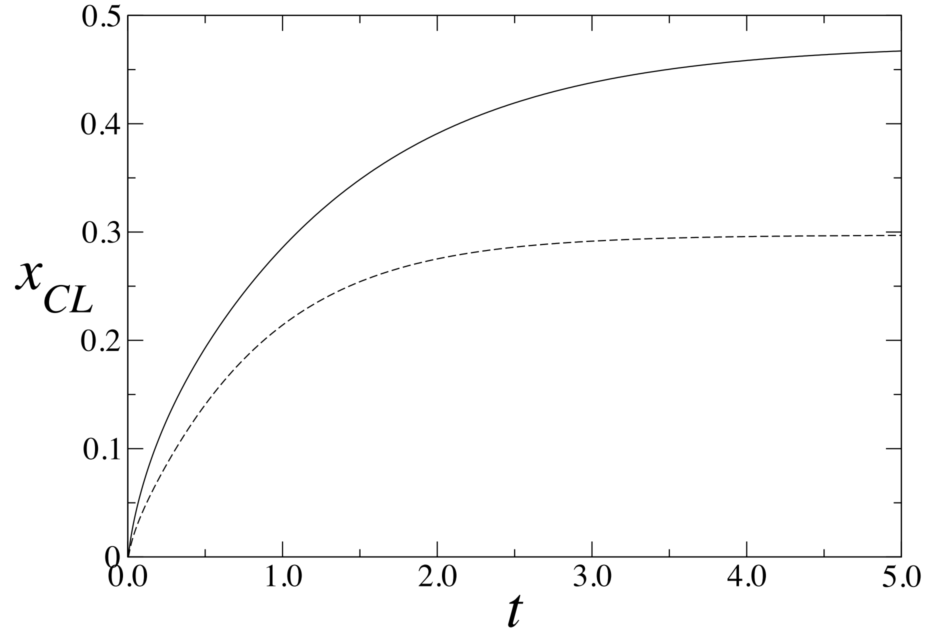



Integrated evaporation rate



# Moving contact lines



# Conclusions

- A lubrication-type model is developed for evaporating meniscus of aqueous solution
- Electrostatic effects are described via nonlinear Poisson-Boltzmann equation and influence *both* normal and tangential interfacial stress conditions
- Curvature and flux profiles near apparent contact line are wider due to electrostatic effects
- Total evaporation rate is increased